Harmonic Magnification by Time Reversal based on a Hammerstein Decomposition

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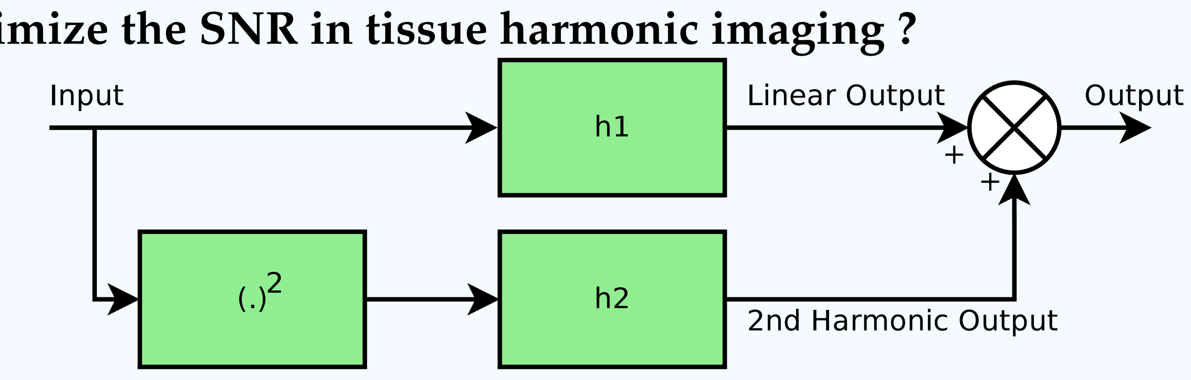
sebastien.menigot@univ-tours.fr, jean-marc.girault@univ-tours.fr

† Université François-Rabelais de Tours, Inserm, Imaging and Brain UMR U930, Tours, France

Introduction

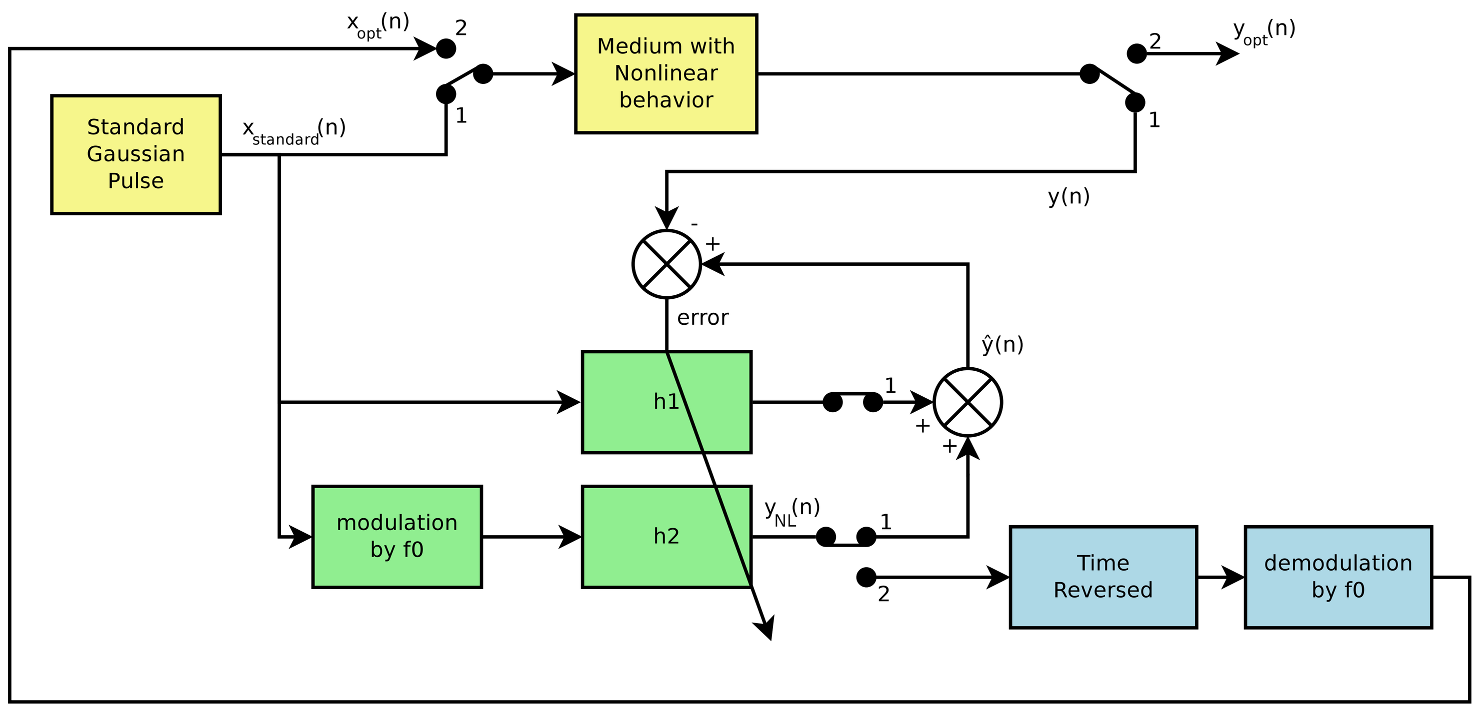
The medical ultrasound imaging systems have been improved by taking into account the nonlinear wave propagation. These improvements leads to increase the signal-to-noise ratio and the focusing in tissue harmonic imaging (THI). Since the backscattered nonlinearities are a function of the transmitted signal, enhancing these nonlinearities means to design the best wave [1]. Time reversal process is well-known to optimize the SNR and free itself to phase aberration, by combining a waveform design in space and in time thanks a physical matched filter [2]. However, it is not well-adjusted for THI, because the propagation of the time reversed signal destroys the harmonics.

How can we optimize the SNR in tissue harmonic imaging ?



As we want to guarantee a SNR optimization with a good focusing, the method has to include time reversal The solution firstly consists in extracting the harmonic component at $2f_0$ by a Hammerstein filter. Then this time reversed harmonic component at $2f_0$ is frequency shifted to the fundamental component at f_0 and retropropagated in the medium.

Methods



1. Sending a first standard excitation $x_{standard}(n)$
2. Harmonic extraction based on a Hammerstein model where the nonlinear function is a frequency shifting by modulation (since it is a bijective function on $\mathbb{R}$):

$$\hat{y}(n) = \sum_{p=1}^{2}\sum_{m=1}^{M} h_p(m)x_{standard}(n-m)\cdot C_p(n), \qquad (1)$$

n the discrete time, M the memory of the Hammerstein model and $C_p(n) = \cos\left(2\pi\frac{(p-1)f_0 n}{F_s}\right)$ with F_s the sample frequency.
The model can be solved by a pseudo-inversion:

$$\mathbf{h} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}. \qquad (2)$$

$\mathbf{h} = [h_1(1),\ldots,h_1(M),h_2(1),\ldots,h_2(M)]$, $\hat{\mathbf{y}} = [y(M+1),\ldots,y(N)]^T$ with N the sample quantity, the matrix of input signals $\mathbf{X} = [\mathbf{x}_1\mathbf{x}_2]$ with

$$\mathbf{x}_p = \begin{pmatrix} v_p(M) & v_p(M+1) & \ldots & v_p(N) \\ v_p(M+1) & v_p(M+2) & & 0 \\ \vdots & & \iddots & \\ v_p(N) & 0 & & 0 \end{pmatrix}, \qquad (3)$$

and $v_p(n) = x(n)C_p(n)$. Finally, the second harmonic signal is: $\mathbf{y}_{NL} = \mathbf{x}_2^T\left[h_2(1),\ldots,h_2(M)\right]^T$.

3. Time reversing $\mathbf{y}_{NL}$
4. Annihilation of the second harmonic effects of the second harmonics:

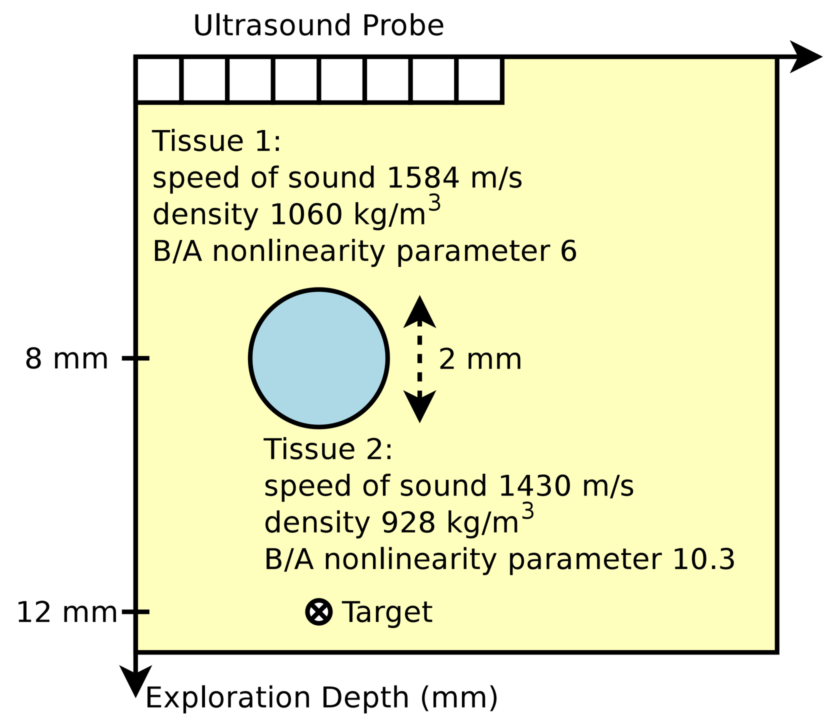
$$x_{opt}(n) = A\cdot y_{NL}(n)\cdot\cos\left(2\pi f_0\frac{n}{F_s}+\phi\right), \qquad (4)$$

A preserve the transmit power to $x_{standard}(n)$ and $\phi = \arctan\left(\frac{\varphi(\mathbf{y}_{NL})}{\varphi(\cos)}\right)$, with $\varphi(\mathbf{y}_{NL})$ the phase of the the second harmonic signal $\mathbf{y}_{NL}$ and the $\varphi(\cos)$ the phase of a cosinus at the frequency $f0$.

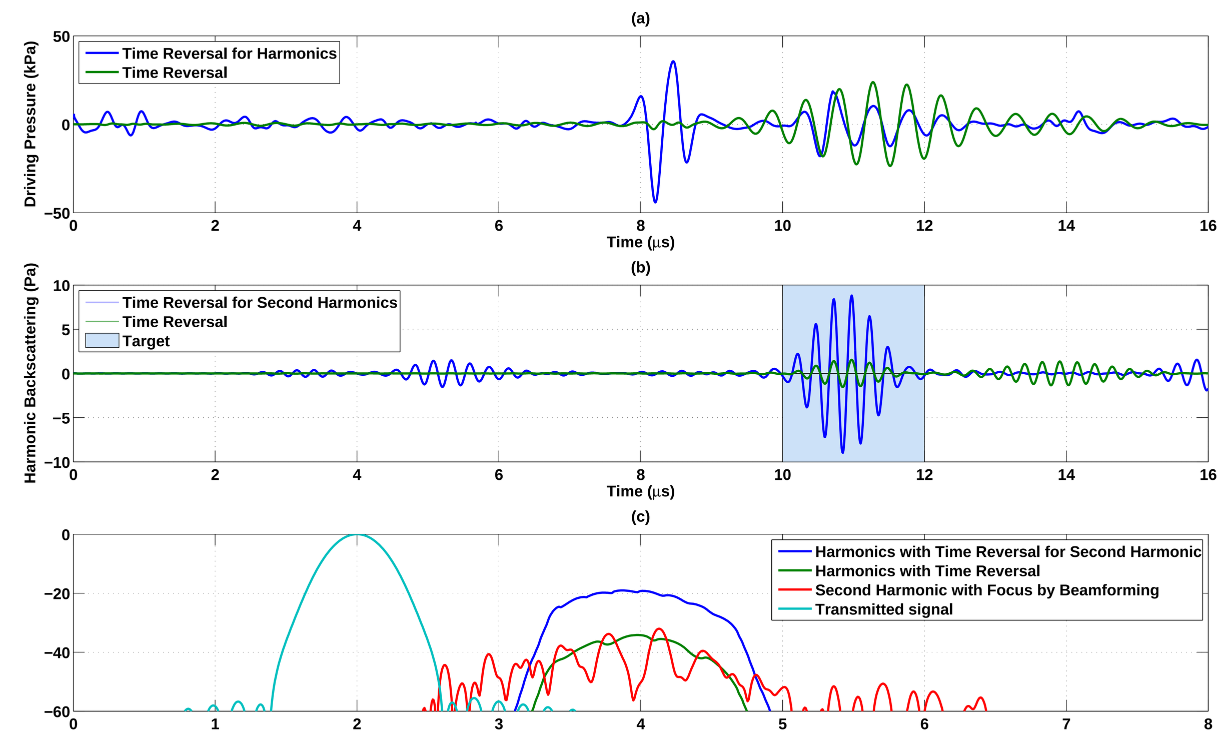
Program example of a Hammertein decomposition on Runmycode.org

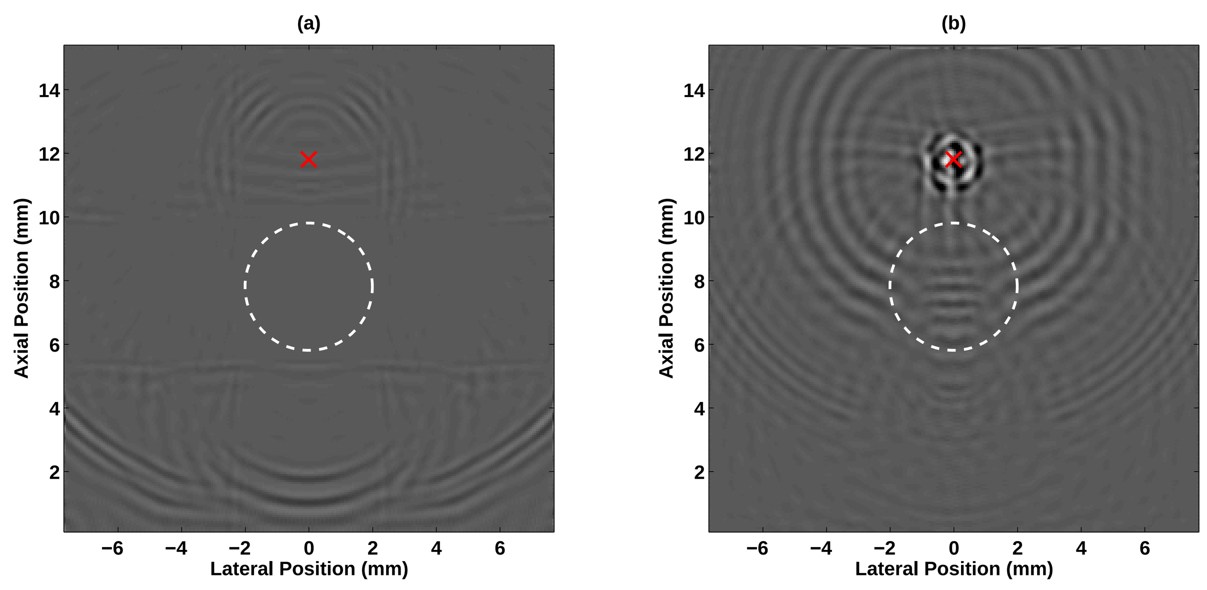
Simulation

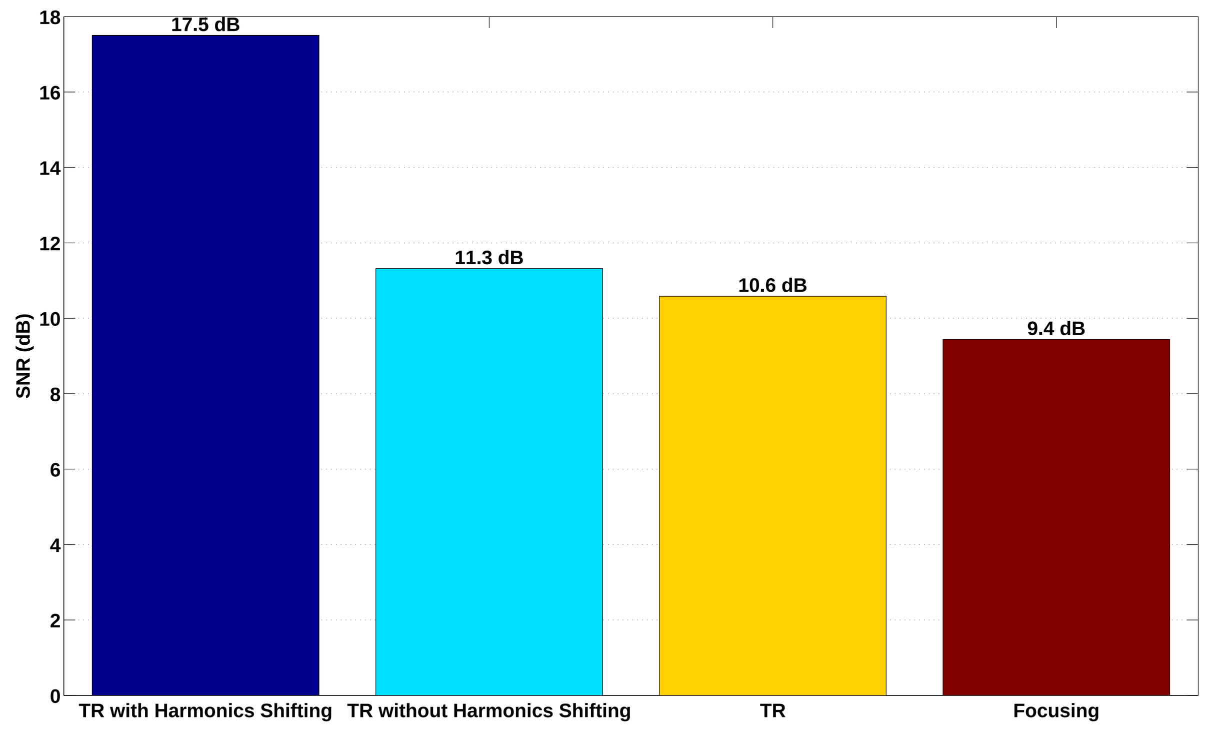
- 2D nonlinear wave propagation in a cavity [3]
- 8-element probe centred at $f_c = 4$ MHz
- Initial Gaussian pulse $x_{standard}(n)$ centred at $f_0 = 22$ MHz and with a bandwidth of 50%



Results







Discussions and Conclusion

- Magnify the second harmonics and improve the SNR
- Hammerstein model using frequency shifting with the good phase
  → Annihilation of the harmonic effects before sending the time-reversed signal
- Matched filter for harmonic components
  → Extension of the time reversal principle to second harmonics

[1] Ménigot S. & Girault, J.-M. *Optimization of Contrast Resolution by Genetic Algorithm in Ultrasound Tissue Harmonic Imaging.* Ultrasonics. **2016**.
[2] Fink, M. *Time-reversal of Ultrasonic Fields. 1. Basic Principles.* IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control. **1992** (39).
[3] Anderson, M. E. *A 2D Nonlinear Wave Propagation Solver Written in Open-Source MATLAB Code.* Proceeding IEEE Ultrasonic Symposium. **2000**.